

# Observation of Ising-like critical fluctuations in frustrated Josephson junction arrays with modulated coupling energies

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## Abstract

We report the results of ac sheet conductance measurements performed on fully frustrated square arrays of Josephson junctions whose coupling energy is periodically modulated in one of the principal lattice directions. Such systems are predicted to exhibit two distinct transitions: a low-temperature Ising-like transition triggered by the proliferation of domain walls and a high-temperature transition driven by the vortex unbinding mechanism of the Beresinskii-Kosterlitz-Thouless (BKT) theory. Both the superfluid and dissipative components of the conductance are found to exhibit features which unambiguously demonstrate the existence of a double transition whose properties are consistent with the Ising-BKT scenario.

*Key words:* Critical phenomena; frustration, domain walls; vortices.

*PACS:* 74.80.F, 74.50, 05.50, 75.30.K

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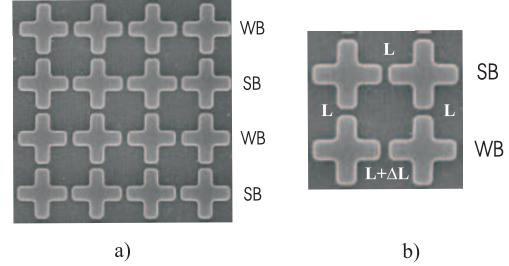
Two-dimensional arrays of Josephson junctions (JJA) exposed to a perpendicular magnetic field provide the opportunity to study the influence of a tunable level of frustration in a variety of topologies ranging from periodic to random structures, including quasiperiodic and fractal lattices. Such systems are a physical realization of the frustrated classical XY model where the degree of frustration is governed by a parameter  $f$  expressing the magnetic flux thread-

ing an elementary cell of the array in units of the superconducting flux quantum.

While the nature of the superconducting transition of a Josephson junction array (JJA) at arbitrary frustrations is still not well understood, the critical behavior of square JJA at full frustration ( $f=1/2$ ) has been widely investigated theoretically and with numerical simulations. Because of the "checkerboard"

structure of the ground state [1], two symmetries are relevant in determining the critical behavior of a fully frustrated JJA: the continuous  $U(1)$  rotational symmetry and the discrete  $Z_2$  chiral symmetry [1–3]. The phase transition resulting by breaking  $U(1)$  is driven, at a temperature  $T_{BKT}$ , by the vortex unbinding mechanism predicted by the Beresinskii-Kosterlitz-Thouless (BKT) theory, whereas the transition associated with  $Z_2$  is triggered by the proliferation of Ising-like domain walls at a temperature  $T_I$ . The question of whether these two transitions are distinct [4] or merge into a single transition belonging to a new universality class [5] has been controversial for a long time. Quite recently Korshunov [6] has shown that the first scenario, with  $T_I > T_{BKT}$ , is the only possible one. Although distinct, the two transitions are, however, very close to each other, making experiments conceived to explore their exact nature rather difficult. From an experimental point of view, the situation is more favorable in frustrated arrays whose coupling energy  $E_J$  is periodically modulated in one of the principal lattice directions. Their critical behavior was first studied with Monte Carlo simulations by Berge et al. [7] and, subsequently, by Eikmans et al. [8] using a Coulomb gas approach. In arrays with a sufficiently strong  $E_J$ -modulation the two transitions are predicted to be well separated (with  $T_I < T_{BKT}$ ) and thus accessible to experimental observation. In this brief report we present preliminary results of ac sheet conductance measurements performed on JJA with modulated

couplings at  $f=1/2$  which unambiguously demonstrate the existence of a double transition with features consistent with the Ising-KT scenario.



a)

b)

Fig. 1. a) SEM picture of a portion of the array showing Pb crosses on a Cu ground plane. The lattice parameter is  $a = 8\mu\text{m}$ . The "weak" JJs compose the first and third lines (WB), whereas the "strong" junctions (SB) compose the second and fourth lines. b) Picture of one square plaquette of the array showing the three "strong" junctions ( $L=0.8\mu\text{m}$ ) and the lengthened junction ( $L+\Delta L=0.9\mu\text{m}$ ).

The experiments were carried out on square arrays of proximity-effect coupled SNS (superconducting-normal-superconducting) Josephson junctions consisting of  $\sim 10^6$  Pb superconducting (S) islands forming a square lattice on a normal (N) Cu layer. To periodically modulate  $E_J$ , the N-bridges of the junctions located on alternating rows were slightly lengthened in order to decrease their coupling energy with respect to that of all the other junctions in the array (see Fig. 1). The lengths of the N-metal gaps of the array shown in Fig. 1 are  $0.8\mu\text{m}$  for the strong bonds (SB) and  $0.92\mu\text{m}$  for the weak bonds (WB). The transition temperature of the S-islands was  $T_{CS} = 7.02\text{K}$  and the normal metal coherence length  $\xi_N(T_{CS}) \cong 80\text{nm}$ . Kinetic inductance measurements performed on

the unfrustrated ( $f=0$ ) array [10] allow to estimate the ratio  $\eta$  between the coupling energies of the weak and the strong bonds. We find  $\eta \cong 0.4$  in the temperature range of interest. The sheet resistance of the array in the normal state was  $R_N \cong 3m\Omega$ .

To explore the properties of the array in the critical region, we measured its complex ac sheet conductance  $G(\omega, T)$  at  $f=1/2$  using a SQUID-operated two-coil mutual inductance technique [9], which allows to probe critical fluctuations over a frequency range covering more than 5 decades (0.1 Hz - 10 kHz). At the temperatures of interest ( $T \cong 5K$ ) we estimate that the vortex diffusion length ( $r_\omega/a \sim (14R_N k_B T/\omega\phi_0^2)^{1/2}$ ) is about  $10^3$  lattice constants at the lowest accessible frequencies. Thus, at these very large length scales, our low-frequency conductance measurements should reflect the response of the array in the quasi-static thermodynamic limit. In this regime therefore the inverse sheet inductance  $L_\square^{-1} = \omega \text{Im}(G)$ , which is proportional to the areal superfluid density and measures the degree of superconducting phase coherence in the system, is directly comparable with Monte Carlo simulations of the array helicity modulus [8].

In Fig. 2 both the superfluid ( $L_\square^{-1}$ ) and the dissipative ( $R=\text{Re}(1/G)$ ) components of the conductance extracted from the linear response of the fully frustrated ( $f=1/2$ ) array at an excitation frequency of 0.23 Hz are shown as a function of temperature on log-lin plots. At  $T \cong 4.8K$  there is a slight depression in  $L_\square^{-1}(T)$

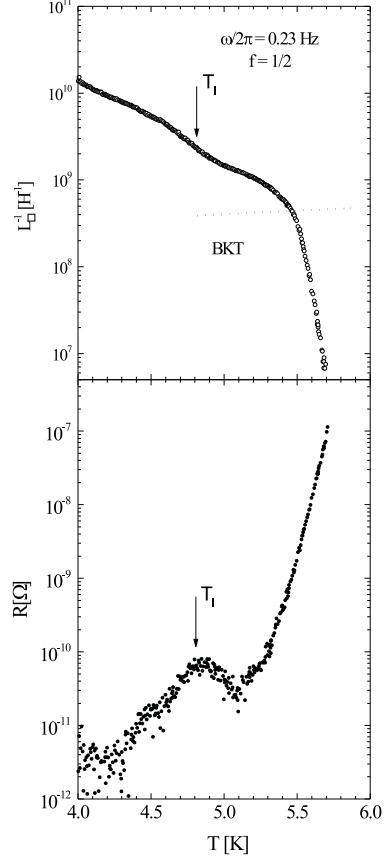


Fig. 2. Inverse sheet inductance ( $L_\square^{-1}$ ) and resistance ( $R$ ) vs temperature at full frustration  $f=1/2$ . The softening of the superconducting phase coherence at  $T_c$  is evidenced by a depression of  $L_\square^{-1}$  and a corresponding shallow peak in  $R$ . The total disappearance of phase coherence and the rapid increase of dissipation are the signatures of the BKT transition. Dotted line: BKT prediction for  $L_\square^{-1}(T_{BKT})$ .

accompanied by a peak in dissipation. Relying on the Coulomb gas analogy [8], we identify these features as the signatures of the "antiferroelectric-paraelectric" Ising transition triggered by the proliferation of domain walls in the system of dipoles created by the attractive interaction, across the weak bonds, of half-integer charges. This inter-

pretation is supported by the observation that, in terms of the reduced temperature  $\tau = k_B T / E_J(T)$ , the transition takes place at  $\tau_I = 0.08$ , in good agreement with the value ( $\tau_I = 0.07$ ) predicted by the phase diagram of Ref.7 for  $\eta \cong 0.4$ . At higher temperatures the superfluid component drops dramatically, signaling the suppression of global superconducting phase coherence, at a temperature  $T_{BKT} \cong 5.5K$ , which is found to be consistent, as shown by the dotted line in Fig. 2, with the universal BKT prediction  $L_{\square}^{-1}(T_{BKT}) = (8\pi/\phi_0^2) k_B T_{BKT}$ .

According to the Coulomb gas analysis [8], the kink structure in  $L_{\square}^{-1}(T)$  at  $T_I$  reflects a logarithmic anomaly in the susceptibility of the Ising-like system of oriented dipoles. More precisely, the temperature derivative  $dL_{\square}^{-1}/dt$  (where  $t = 1 - T/T_I$ ) should exhibit, at  $T_I$ , a  $\ln|t|$  divergence, which can be studied in great detail by varying the length scale (i.e. the driving frequency  $\omega$ ) at which one is probing the JJA. The results of these investigations and, more generally, of the very interesting frequency dependence of the dynamic response in the critical Ising-BKT region will be published elsewhere.

We would like to thank S.E. Korshunov for useful discussions. This work was supported by the Swiss National Science Foundation, the Swiss Federal Office for Education and Science within the framework of the TMR network "Superconducting Nanocircuits" of the European Union and the Vortex Program of the European Science Foundation.

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